Nonlinear Pulse Propagation and Phase Velocity of Laser-Driven Plasma Waves

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Laser evolution and plasma wave excitation by a relativistically intense short-pulse laser in underdense plasma are investigated in the broad pulse limit, including the effects of pulse steepening, frequency redshifting, and energy depletion. The nonlinear plasma wave phase velocity is shown to be significantly lower than the laser group velocity and further decreases as the pulse propagates owing to laser evolution. This lowers the thresholds for trapping and wave breaking and reduces the energy gain and efficiency of laser-plasma accelerators that use a uniform plasma profile.

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The propagation velocity of laser pulses and the phase velocity of plasma waves are of fundamental importance to many areas of plasma physics. For example, in laser-plasma accelerators (LPAs) [1], which have demonstrated the production of high-quality GeV electron beams in centimeter-scale plasmas [2], the dynamics of the accelerated electrons is strongly affected by the plasma wave phase velocity. The phase velocity determines the dephasing length (distance for a relativistic particle to move out of an accelerating phase) and, hence, the maximum energy gain of the electrons in the plasma wave [3], as well as the trapping threshold for background plasma electrons [4] and the maximum amplitude of the plasma wave [5]. The plasma wave phase velocity driven by a short-pulse laser is intrinsically related to the drive laser velocity and evolution. A calculation of these velocities is essential for the design and understanding of present and future LPA experiments.

In LPAs, the electron plasma wave is driven by the laser ponderomotive force. For low laser intensities $a_0^2 \ll 1$, the phase velocity of the plasma wave is approximately the group velocity of the laser. Here $a_0^2 \simeq 7.32 \times 10^{-19} \lambda_0^2[\mu\text{m}] I_0[\text{W/cm}^2]$ for a linearly polarized laser with $\lambda_0 = 2\pi/k_0 = 2\pi c/\omega_0$ the laser wavelength and I_0 the peak intensity. For a low-intensity laser pulse propagating in a uniform, underdense plasma ($\omega_p^2/\omega_0^2 \ll 1$), the linear laser group velocity is $v_g/c = \beta_g \simeq 1 - \omega_p^2/2\omega_0^2$ in the one-dimensional (1D) limit, where $\omega_p = k_p c = 2\pi c/\lambda_p = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency, with n_0 the unperturbed neutral plasma number density, m the electron mass, e the electron charge, and c the speed of light in vacuum. In the linear regime the plasma wave phase velocity is $v_p \simeq v_g$ with the Lorentz factor $\gamma_p \simeq \gamma_g = (1 - \beta_g^2)^{-1/2} = \omega_0/\omega_p$.

Present LPA experiments operate at relativistic intensities $I_0 > 10^{18}$ W/cm^2, or $a_0 \gtrsim 1$, to drive large amplitude plasma waves for particle acceleration. In the nonlinear regime, calculations of the plasma wave phase velocity are generally lacking in the literature. Some approximate expressions have been calculated in limited regimes. Lu *et al.* [6] used particle-in-cell simulations to estimate a constant phase velocity $\gamma_p = \omega_0/\sqrt{3}\omega_p$ in the blowout regime ($a_0 \sim 4$) and a phase velocity of $\gamma_p = \sqrt{a_0}\omega_0/\omega_p$ in the nonlinear 1D regime. Earlier work, also using particle-in-cell simulations, showed $v_p < v_g$ in the nonlinear 1D regime [7]. More typically, the approximation $v_p \simeq v_g$ is currently used in the literature, with v_g approximated by the linear group velocity. As shown in this work, this is a poor approximation in the nonlinear regime $a_0 \gtrsim 1$. In general, the wave phase velocity is determined by the nonlinear laser intensity transport velocity and laser evolution. In addition, the wave phase velocity evolves as the laser propagates and deposits energy into the plasma wave over a depletion length [8].

In this Letter, we investigate the propagation of high-intensity ($a_0 \sim 1$) laser pulses in 1D in underdense plasma and calculate the nonlinear intensity transport and group velocities of the laser pulse and the nonlinear phase velocity of the excited plasma wave. These are calculated by using a reduced wave equation and are shown to be in agreement with solutions to the full Maxwell equations coupled to the nonlinear plasma fluid response.

Laser propagation is considered in a cold, collisionless, underdense plasma (with immobile ions). It is convenient to introduce the normalized vector potential $\boldsymbol{a} = e\boldsymbol{A}/mc^2$ (in the Coulomb gauge $\nabla \cdot \boldsymbol{a} = 0$), the normalized space-charge potential $\phi = e\Phi/mc^2$, and the proper density $\rho = (n/n_0)\gamma^{-1}$, where n is the plasma density and $\gamma = \sqrt{1 + u^2}$ is the Lorentz factor of the normalized fluid momentum $\boldsymbol{u} = \boldsymbol{p}/mc$. Introducing the variables $\zeta = z - ct$ and $\tau = ct$, and the laser pulse envelope $\hat{a}$ as $\boldsymbol{a}_\perp = (\hat{a}/2)\exp(ik_0\zeta)\hat{\boldsymbol{e}}_\perp + \text{c.c.}$, the wave equation for the laser field becomes [1]

$$[\nabla_\perp^2 + 2(ik_0 + \partial_\zeta)\partial_\tau - \partial_\tau^2]\hat{a} = k_p^2 \rho \hat{a}. \tag{1}$$

Assuming $k_p \ll k_0$, associated with Eq. (1) is the adiabatic invariant (the wave action $\mathcal{A}$) [9], such that

$$\partial_\tau \mathcal{A} = \partial_\tau \int d^2\boldsymbol{x}_\perp \int k_p d\zeta \hat{a}^*[1 - ik_0^{-1}(\partial_\zeta - \partial_\tau)]\hat{a} = 0. \quad (2)$$

By using Eq. (1), the laser energy evolution [10] is

$$\partial_\tau \mathcal{E} = (k_p/k_0)^2 \int d^2\boldsymbol{x}_\perp \int k_p d\zeta \rho \partial_\zeta |\hat{a}|^2/2, \quad (3)$$

where $\mathcal{E} = \int d^2\boldsymbol{x}_\perp \int k_p d\zeta |[1 - ik_0^{-1}(\partial_\zeta - \partial_\tau)]\hat{a}|^2$ is the normalized laser energy.

For short-pulse ($k_p L \sim 1$) interactions in underdense plasma ($k_p \ll k_0$), $|\partial_\zeta \hat{a}| \sim |\hat{a}|/L \sim k_p|\hat{a}| \ll k_0|\hat{a}|$, $|\nabla_\perp \hat{a}| \sim |\hat{a}|/r_0$, and $|\partial_\tau \hat{a}| \sim |\hat{a}|/L_e$, where r_0 is the spot size, L is the pulse length, and L_e is the characteristic length for laser evolution. We assume a sufficiently broad pulse such that the transverse operator may be neglected. In this regime $L_e \sim k_0^2/k_p^3$, $|\partial_\tau \hat{a}| \ll |\partial_\zeta \hat{a}| \ll |k_0 \hat{a}|$, and the wave equation (1) can be approximated by coupled equations for the laser intensity and wave number:

$$\partial_{\hat{\tau}} a^2 = \hat{k}^{-2}[a^2\rho' + (\rho a^2)' - 2a^2\rho \hat{k}^{-1}\hat{k}']/2, \quad (4)$$

$$\partial_{\hat{\tau}} \hat{k} = \hat{k}^{-2}[\rho \hat{k}' - \hat{k}\rho']/2, \quad (5)$$

where $\hat{\tau} = k_p^3 \tau / k_0^2$, the primes denote $k_p^{-1}\partial_\zeta$, $\hat{k} = k/k_0 = 1 + k_0^{-1}\partial_\zeta \theta$ is the laser wave number normalized to the initial wave number, and $\hat{a} = a\exp(i\theta)$. Equation (4) describes pulse steepening, and Eq. (5) describes self-phase modulation and frequency shifts. Assuming the laser is initially monochromatic without a chirp, then for early times ($\hat{\tau} < 1$), Eqs. (4) and (5) simplify to $\partial_{\hat{\tau}} a^2 = a^2\rho' + \rho a a'$ and $\partial_{\hat{\tau}} \hat{k} = -\rho'/2$, respectively (these equations will be used to describe the early-time evolution in the following).

For $L \ll L_e$, the quasistatic approximation [1] may be applied to the plasma fluid equations. Assuming a broad laser pulse $k_p r_0 \gg 1$, the quasistatic fluid equations reduce to

$$x'' = (\gamma_\perp^2 x^{-2} - 1)/2, \quad (6)$$

where $x = 1 + \phi$, $\rho = 1/x$, $-x' = -k_p^{-1}\partial_\zeta \phi$ is the plasma wave electric field normalized to $E_0 = mc^2 k_p/e$, and $\gamma_\perp^2 = 1 + a^2/2$. Behind the laser ($\gamma_\perp = 1$), the first integral of Eq. (6) yields $x'^2 + x + x^{-1} = \hat{E}_m^2 + 2$, where $\hat{E}_m = E_{\text{peak}}/E_0$ is the peak field behind the laser.

By using the first integral of Eq. (6), the rate of change of the integrated normalized intensity $\mathcal{Q} = \int k_p d\zeta a^2$ for early times is

$$\partial_{\hat{\tau}} \mathcal{Q} = \int k_p d\zeta \partial_{\hat{\tau}} a^2 = \int d\zeta a^2 \partial_\zeta \rho/2 = \hat{E}_m^2. \quad (7)$$

This describes the early-time steepening of a resonant laser pulse. From Eq. (3) in 1D, $\partial_{\hat{\tau}} \mathcal{E} = -\hat{E}_m^2 = -\partial_{\hat{\tau}} \mathcal{Q}$; i.e., the rate of energy depletion [8] is equal to the rate of pulse steepening. The mean laser wave number can be expressed as $\langle k/k_0 \rangle = \mathcal{E}/\mathcal{A}$. From action conservation $\partial_{\hat{\tau}} \mathcal{A} = 0$, as the pulse steepens, the mean wave number decreases (redshifts) and the laser energy depletes:

$$\mathcal{A}\, \partial_{\hat{\tau}} \langle k/k_0 \rangle = \partial_{\hat{\tau}} \mathcal{E} = -\partial_{\hat{\tau}} \mathcal{Q} = -\hat{E}_m^2, \quad (8)$$

and the rate is determined by the peak of the accelerating field behind the laser $\hat{E}_m$.

At the phase of the peak field ζ_p, $x''(\zeta_p) = 0$, and the evolution of the peak electric field is

$$\partial_{\hat{\tau}} \hat{E}_m = \partial_{\hat{\tau}} E(\zeta_p)/E_0 = -\partial_{\hat{\tau}} x'(\zeta_p) = -y'(\zeta_p), \quad (9)$$

with $y = \partial_{\hat{\tau}} \phi = \partial_{\hat{\tau}} x$. By using Eq. (6), $y'' = -\gamma_\perp^2 x^{-3} y + x^{-2}\partial_{\hat{\tau}} a^2/4$. Substituting the early-time evolution of the laser intensity yields

$$y'' = -\gamma_\perp^2 y x^{-3} - [a^2 x^{-4} x' - x^{-3} a a']/4. \quad (10)$$

Equations (6) and (10) can be solved for the rate of change of the plasma wave amplitude and hence the laser evolution Eq. (8). In the limit of $a_0^2 \ll 1$, $\hat{E}_m/\hat{E}_m(0) \simeq 1 + 0.2a_0^2\hat{\tau}$ for a resonant Gaussian pulse $k_p L = 1$ with $\hat{a}(\zeta) = a_0 \exp(-\zeta^2/4L^2)$.

By using Eq. (9), $\hat{E}_m^2 \simeq \hat{E}_m^2(0)[1 + 2\hat{\tau} y'(\zeta_p)/x'(\zeta_p)]$, and the evolution of the laser energy is

$$\mathcal{E}/\mathcal{E}_0 = 1 - \hat{\tau}/\hat{L}_{\text{pd}} - \hat{\tau}^2 y'(\zeta_p) x'(\zeta_p)/\mathcal{E}_0. \quad (11)$$

Here $\hat{L}_{\text{pd}}$ is the initial normalized depletion length [8] $\hat{L}_{\text{pd}} = k_p^3 L_{\text{pd}}/k_0^2 = \mathcal{E}_0/\hat{E}_m^2(0)$, where $\mathcal{E}_0$ is the initial pulse energy ($\mathcal{E}_0 = a_0^2\sqrt{2\pi} k_p L$ for a Gaussian profile).

The laser intensity centroid (i.e., position weighted by $|\hat{a}|^2$) may be defined as $\bar{\zeta} = \mathcal{Q}^{-1} \int k_p d\zeta a^2 \zeta$, and the laser intensity transport velocity β_I is given by $\partial_\tau \bar{\zeta} = \delta\beta_I = \beta_I - 1$. By using the evolution equation for the intensity, the early-time intensity transport velocity is

$$\partial_{\hat{\tau}} \bar{\zeta} = \mathcal{Q}^{-1} \int k_p d\zeta (\zeta - \bar{\zeta})(\rho' a^2 + \rho a a'), \quad (12)$$

with $\rho = 1/x$ in the quasistatic approximation. In the long-pulse, adiabatic limit, $x = \gamma_\perp$, $x' = 0$, and Eq. (12) yields the Lorentz factor $\gamma_I = (1 - \beta_I^2)^{-1/2} = (k_0/k_p) \times [\gamma_\perp(\gamma_\perp + 1)/2]^{1/2}$. For $|\hat{a}|^2 \ll 1$, $\gamma_I \simeq (1 + 3|\hat{a}|^2/16) \times (k_0/k_p)$ in the adiabatic limit, which agrees with the result in Ref. [11]. Evaluating Eq. (12) for a resonant sine pulse yields $\beta_I = 1 + \partial_\tau \bar{\zeta} = 1 - (k_p/k_0)^2 \times \{1 - [(75 - 2\pi^2)/192]a_0^2\}/2$. For a resonant Gaussian pulse, evaluation of Eq. (12) yields the Lorentz factor $\gamma_I \simeq (1 + 0.10a_0^2)(k_0/k_p)$. The Lorentz factor of the laser intensity transport velocity grows linearly with laser intensity for $a_0^2 \ll 1$, with the coefficient determined by the specific laser profile.

The laser intensity transport velocity Eq. (12) differs from the nonlinear laser group velocity [7]. The laser group velocity defined as the velocity of the laser energy centroid $\langle \zeta \rangle$ is $\beta_g = 1 + \partial_\tau \langle \zeta \rangle$ with

$$\partial_{\hat{\tau}}\langle\zeta\rangle = \mathcal{E}^{-1}\int k_p d\zeta(\zeta - \langle\zeta\rangle)\rho a a'. \quad (13)$$

For a resonant sine-pulse profile with $a_0^2 \ll 1$, evaluating Eq. (13) yields $\partial_{\hat{\tau}}\langle\zeta\rangle = -\{1 - [(15 + 2\pi^2)/192]a_0^2\}/2$. For a resonant Gaussian pulse, evaluation of Eq. (13) yields the group velocity Lorentz factor $\gamma_g \simeq (1 + 0.088a_0^2) \times (k_0/k_p)$ for $a_0^2 \ll 1$.

The plasma wave phase velocity is determined by the intensity transport velocity and the evolution of the laser. The nonlinear phase velocity of the plasma wave, defined as the velocity of the peak of accelerating field, can be obtained from Eqs. (4)–(6). The peak of the electric field behind the laser occurs at a phase ζ_p such that $\partial_\zeta^2\phi(\zeta_p) = 0$, i.e., from Eq. (6), at $\phi(\zeta_p) = 0$. The velocity of the peak field is determined by the evolution of $\zeta_p(\tau)$, i.e., $\beta_p = 1 + \delta\beta_p = 1 + [\zeta_p(\Delta\tau) - \zeta_p(0)]/\Delta\tau$. For the peak field, $\phi(\zeta_p(\Delta\tau), \Delta\tau) = \phi(\zeta_p(0), 0) = 0$, and the phase velocity is given by

$$\delta\beta_p = -\partial_\tau\phi(\zeta_p)/\partial_\zeta\phi(\zeta_p). \quad (14)$$

Equations (6) and (10) can be solved for the initial nonlinear phase velocity $\beta_p = 1 - [y(\zeta_p)/x'(\zeta_p)](k_p/k_0)^2$ given the laser envelope $a(\zeta)$. Because of the evolution of the shape of the field, the phase velocity is dependent on the phase in the wave. The phase velocity of the zero crossing of the field, where $x'(\zeta_0) = 0$, is $\beta_p(\zeta_0) = 1 - \partial_{\zeta\tau}^2\phi(\zeta_0)/\partial_\zeta^2\phi(\zeta_0) = 1 - [y'(\zeta_0)/x''(\zeta_0)](k_p/k_0)^2$.

Figure 1 shows the Lorentz factor of the initial phase velocity γ_p, laser group velocity γ_g, and intensity transport velocity γ_I versus a_0 for a resonant ($k_pL = 1$) Gaussian laser profile $\hat{a}(\zeta) = a_0\exp(-\zeta^2/4L^2)$. The curves are the analytic calculations using Eqs. (12)–(14), and the points are solutions to the full Maxwell-fluid equations (with $k_0/k_p = 20$) using the code INF&RNO [12]. As Fig. 1 illustrates, modeling the phase velocity as the linear laser group velocity $(k_p/k_0)\gamma = 1$ (or nonlinear laser velocities γ_g or γ_I) can be a poor approximation. The second (and subsequent) plasma wave periods have lower phase velocities owing to the increasing nonlinear plasma wavelength as the intensity grows (i.e., pulse steepening).

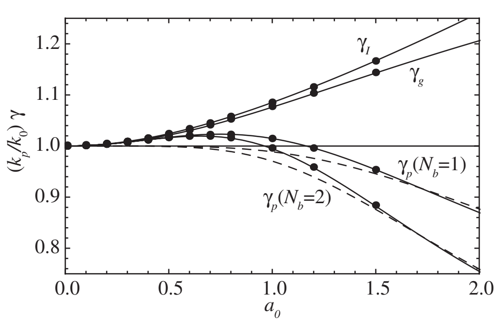

FIG. 1. Lorentz factor of the phase velocity $(k_p/k_0)\gamma_p$ of the peak field (first $N_p = 1$ and second $N_p = 2$ plasma wave periods), intensity transport velocity $(k_p/k_0)\gamma_I$, and group velocity $(k_p/k_0)\gamma_g$ for a resonant Gaussian laser vs a_0. Curves are analytic theory, and points are solutions to full Maxwell-fluid equations (with $k_0/k_p = 20$). Dashed curves are the phase velocity due to the plasma wavelength evolution Eq. (15).

For $a^2 \ll 1$, Eq. (6) may be expanded to order $\mathcal{O}(a^6)$ and solved perturbatively to yield the initial phase velocity. For an initially resonant Gaussian pulse, the phase velocity of the peak field in the first wave period following the laser is $\gamma_p \simeq (k_0/k_p)[1 + 0.10a_0^2 - 0.12a_0^4 + 0.05a_0^6]$. To order $\mathcal{O}(a^2)$, the phase velocity is approximately the intensity transport velocity $\gamma_p \simeq \gamma_I$. Higher-order nonlinearities are due to the evolution of the plasma wave driven by an evolving laser, i.e., changes in the nonlinear plasma wavelength and the phase location of peak field.

For $a_0^2 > 1$, the phase velocity is initially dominated by the nonlinear increase in the plasma wavelength and wave amplitude owing to the laser steepening. For $\gamma_p^2 \gg 1$, the nonlinear plasma wavelength is [1] $\lambda_{Np} = 2\pi/k_{Np} = (2/\pi)\lambda_p x_m^{1/2}E_2(1 - 1/x_m^2)$, where E_2 is the complete elliptic integral of the 2nd kind, and $x_m = 1 + \hat{E}_m^2/2 + [(1 + \hat{E}_m^2/2)^2 - 1]^{1/2}$ is the maximum potential. Assuming the plasma wave phase is a function of $k_{Np}\zeta$, the contribution to the phase velocity owing to the evolution of λ_{Np} is

$$\delta\beta_p \simeq \zeta_p\lambda_{Np}^{-1}(\partial\lambda_{Np}/\partial\hat{E}_m)\partial_\tau\hat{E}_m, \quad (15)$$

which can be evaluated by using Eq. (9). The dashed curves in Fig. 1 show the phase velocity given by Eq. (15) for plasma wave periods $N_p = 1$ and 2. In the ultrarelativistic limit $a_0 \gg 1$, the nonlinear phase velocity driven by a Gaussian laser (with $k_pL = 1$) asymptotes to $\gamma_p \approx 0.45N_p^{-1/2}(k_0/k_p)$.

As the laser evolves, so do the plasma wave and its phase velocity. Figure 2 shows the evolution of the phase velocity (solid curves in Fig. 2 are solutions to the full Maxwell-fluid equations). The decrease in phase velocity in time is due to the laser frequency redshifting and the pulse steepening (growing plasma wavelength). The rate of decrease

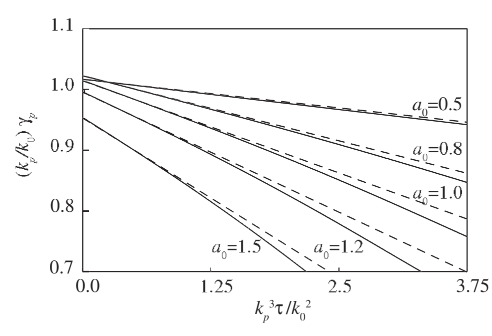

FIG. 2. Lorentz factor of the phase velocity $(k_p/k_0)\gamma_p$ of the peak field in the first plasma wave bucket vs $\tau k_p^3/k_0^2$ for initial intensities $a_0 = 0.5$, 0.8, 1.0, 1.2, and 1.5 and for an initially resonant Gaussian laser with $k_0/k_p = 20$. Dashed lines show the initial rate of change of the phase velocity Eq. (16).

of the phase velocity is approximately linear, and the initial rate can be calculated by using

$$\partial_\tau \delta\beta_p(\zeta_p) = \frac{-\partial_\tau^2 \phi[\zeta_p(0)]}{\partial_\zeta \phi[\zeta_p(0)]} + \frac{2\partial_\tau \phi[\zeta_p(0)]\partial_{\zeta\tau}^2 \phi[\zeta_p(0)]}{\{\partial_\zeta \phi[\zeta_p(0)]\}^2} \quad (16)$$

for the peak of the field, or $\partial_\tau \delta\beta_p(\zeta_p) = [-w/x' + 2yy'/(x')^2](k_p/k_0)^4$, where $w = \partial_{\hat{\tau}}^2 \phi$. By using Eq. (10),

$$w'' = -\gamma_\perp^2(wx^{-3} - 3y^2x^{-4}) - yx^{-3}\partial_{\hat{\tau}} a^2 + x^{-2}\partial_{\hat{\tau}}^2 a^2/4, \quad (17)$$

where $\partial_{\hat{\tau}}^2 a^2$ is given by Eqs. (4) and (5) with the initial conditions, e.g., $\hat{k}(0) = 1$, $\hat{k}'(0) = 0$, $\partial_{\hat{\tau}}\hat{k}(0) = x'/2x^2$, and $\partial_{\hat{\tau}} a^2(0) = -x'a^2/x^2 + aa'/x$. The rate of change of the phase velocity is given by the coupled equations (6), (10), and (17). Figure 2 shows a comparison of the rate of change of the phase velocity Eq. (16) (dashed curves) and the solutions to the full Maxwell-fluid equations (solid curves). For an initially resonant Gaussian pulse with $a_0^2 < 1$, the initial rate of change of the Lorentz factor of the phase velocity (of the peak field in the second plasma wave bucket) is $(k_p/k_0)\partial_{\hat{\tau}}\gamma_p \simeq -0.09a_0^2 + 0.03a_0^4$.

The dephasing length L_d can be defined as the propagation distance required for a particle moving at $\simeq c$ to travel from the peak of the field, at phase ζ_p such that $x(\zeta_p) = 1$, to the zero crossing of the field, at phase ζ_0 such that $x'(\zeta_0) = 0$. This distance will be determined by the phase velocity of the zero crossing $\beta_p(\zeta_0)$. The normalized dephasing length $\hat{L}_d = k_p^3 L_d/k_0^2$ is given by $\zeta_p(0) = \zeta_0(\hat{L}_d) = \zeta_0(0) + (-y'/x'')\hat{L}_d + (2y'y''/x''^2 - w'/x'')\hat{L}_d^2/2$. Figure 3 shows the nonlinear dephasing length $\hat{L}_d$ for $N_p = 2$ versus a_0 assuming a resonant Gaussian laser. Also shown is the energy remaining in the laser at the dephasing length $\mathcal{E}(L_d)/\mathcal{E}_0$ given by Eq. (11). The points in Fig. 3 are solutions to the full Maxwell-fluid equations. As the intensity increases, L_d is reduced owing to the decreasing phase velocity, and the laser does not efficiently deplete its energy over this reduced interaction length. This indicates the necessity of tapering [13,14] (i.e., an increasing plasma density such that the plasma wavelength shortens, compensating for the particle slippage) for efficient laser-plasma accelerators.

In this Letter, we have investigated the evolution of an intense ($a_0 \sim 1$) short-pulse ($k_p L \sim 1$) laser in an underdense ($k_p/k_0 \ll 1$) plasma and of the excited plasma wave. Expressions governing the laser evolution (depletion, pulse steepening, redshifting, and laser velocities) and plasma wave evolution (wave amplitude and phase velocity) were derived by using a reduced wave equation coupled to the quasistatic plasma response and were found to be in good agreement with the full Maxwell-fluid equations. This work assumed a 1D, broad pulse limit, which will be valid for a laser spot size r_0 such that $k_p^2 r_0^2 > 1$ and $a^2/\gamma_\perp < k_p^2 r_0^2$. In the nonlinear regime $a_0 \gtrsim 1$, the nonlinear plasma

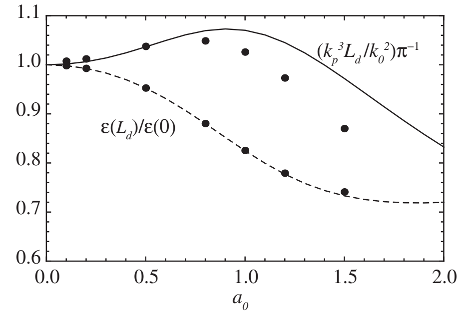

FIG. 3. Normalized dephasing length (solid curve) vs a_0, for a relativistic beam in the second plasma wave bucket. Fraction of laser energy (dashed curve) remaining after a dephasing length. Points are solutions to full Maxwell-fluid equations.

wave phase velocity is substantially lower than the laser group velocity. The phase velocity is lower in subsequent plasma wave periods owing to the evolution (lengthening) of the nonlinear plasma wavelength. As the laser propagates, pulse evolution (redshifting and steepening) further decreases the phase velocity. This indicates that electron trapping and wave breaking of plasma waves will occur at lower thresholds than estimations based on the linear group velocity. Furthermore, this indicates that electron dephasing (rather than laser depletion) will limit the energy gain in an LPA that uses an axially uniform plasma. This implies that plasma tapering is necessary to enhance the single-stage energy gain and efficiency of LPAs.

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