Warm wave breaking of nonlinear plasma waves with arbitrary phase velocities

C. B. Schroeder, E. Esarey, and B. A. Shadwick
Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
(Received 12 November 2004; published 22 November 2005)

A warm, relativistic fluid theory of a nonequilibrium, collisionless plasma is developed to analyze nonlinear plasma waves excited by intense drive beams. The maximum amplitude and wavelength are calculated for nonrelativistic plasma temperatures and arbitrary plasma wave phase velocities. The maximum amplitude is shown to increase in the presence of a laser field. These results set a limit to the achievable gradient in plasma-based accelerators.

DOI: 10.1103/PhysRevE.72.055401 PACS number(s): 52.27.Ny, 52.38.−r, 52.38.Kd, 52.35.Mw

Of fundamental interest in plasma physics are highly nonlinear electron plasma waves, such as those produced in the laboratory via intense laser and beam plasma interactions [1]. Recent plasma-based accelerator experiments [2–4] have shown the production of high-quality electron bunches using ultrahigh gradient (~100 GV/m, several orders of magnitude beyond conventional technology) nonlinear plasma waves driven by intense laser pulses. A basic quantity of interest in plasma physics, and especially relevant to plasma accelerators, is the maximum achievable plasma wave amplitude (the wave-breaking limit). Prior calculations [5–9] of the wave-breaking limit, however, are not valid in the regime of laser-plasma accelerator experiments.

In this paper, a general result for the maximum field amplitude of a nonlinear electron plasma wave of arbitrary phase velocity in a warm plasma is derived from first principles. This result is valid in all regimes of interest, including that of short-pulse laser-plasma interactions, and reduce to the previous wave-breaking calculations [5–9] in the appropriate limits. The effects of an intense laser field are also included, as in the self-modulated regime of the laser wakefield accelerator [10–12], which is shown to increase the maximum field amplitude. The maximum field amplitude sets the fundamental limit to the achievable gradient in plasma-based accelerators.

Using the cold, relativistic fluid equations in one dimension, the maximum electric field amplitude of a plasma wave was found [5] to be $E_{\mathrm{WB}}=\sqrt{2}(\gamma_\varphi-1)^{1/2}E_0$, which is referred to as the cold relativistic wave-breaking field. Here $\gamma_\varphi^2=1/(1-\beta_\varphi^2)$ is the relativistic Lorentz factor, $v_\varphi=c\beta_\varphi$ is the plasma wave phase velocity, and $E_0=cm\omega_p/e$ is referred to as the nonrelativistic wave-breaking field, with $\omega_p=(4\pi n_0 e^2/m)^{1/2}$ the plasma frequency and n_0 the ambient electron plasma density. For a laser driven plasma wave, v_φ is approximately the group velocity of the laser pulse, $\gamma_\varphi\simeq\omega_0/\omega_p$, where ω_0 is the laser frequency. For a charged particle beam driver, v_φ is approximately the particle beam velocity. When the plasma wave field approaches E_{WB}, the cold plasma density becomes singular $n\to\infty$ [6]. This singularity indicates a breakdown of the cold fluid equations.

Finite temperature fluid theories were applied to calculate the maximum amplitudes in the limits of nonrelativistic ($\gamma_\varphi\simeq 1$) [7] and ultrarelativistic ($\beta_\varphi=1$) [8,9] plasma waves. In the $\beta_\varphi=1$ limit, the maximum field was found [8,9] to be $E_{\mathrm{th}}=\theta^{-1/4}\rho_{\mathrm{th}}(\gamma_\varphi,\theta)E_0$, where θ is the initial plasma temperature normalized to mc^2/k_{B}, with k_{B} the Boltzmann constant, and $\rho_{\mathrm{th}}(\gamma_\varphi,\theta)\sim 1$ is a slowly varying function of γ_φ and θ. This expression for E_{th} is valid for $\gamma_\varphi\theta^{1/2}\gg 1$, e.g., for an ultrarelativistic ($\beta_\varphi=1$) particle beam driver. For laser-driven plasma waves, however, typically $\gamma_\varphi\sim 10-100$ and $\theta mc^2\sim 10$ eV [13,14]. Therefore, a laser-plasma accelerator typically satisfies $\gamma_\varphi\theta^{1/2}<1$, and, hence, the above expression for E_{th} does not apply. Low-energy particle beam-driven plasma waves also satisfy $\gamma_\varphi\theta^{1/2}<1$, such as those produced in the first plasma wake-field accelerator experiments [15]. In addition, E_{th} does not reduce to the nonrelativistic result [7] or the cold result E_{WB}.

Standard warm relativistic fluid theories derived for collisionally dominated plasmas (e.g., Ref. [16]) are inadequate for describing short-pulse laser-plasma interactions. Short-pulse laser-plasma interactions access a collisionless regime that is not in local thermodynamical equilibrium, in which the plasma electrons experience relativistic motion while the temperature (electron momentum spread) remains small. To model short-pulse laser-plasma interactions, we start with the covariant form of the collisionless Boltzmann equation [16],

$$p^\mu\partial_\mu f-[(e/mc^2)F^{\alpha\nu}p_\nu]\partial f/\partial p^\alpha=0, \tag{1}$$

where $f(\mathbf{x},\mathbf{p},t)$ is the phase-space density, $x^\mu=(ct,\mathbf{x})$, $p^\nu=(\gamma,\gamma\boldsymbol{\beta})$ is the normalized particle four momentum, $\partial^\mu=(\partial_{ct},-\nabla)$, and $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$ is the electromagnetic field-strength tensor, with $A^\mu=(\Phi,\mathbf{A})$ the four-vector potential, and $g^{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$ the space-time metric tensor.

We consider the following centered moments of the phase-space distribution [17–19]: $\Theta^{\mu\nu}=\int(p^\mu-u^\mu)(p^\nu-u^\nu)fd\Omega$ and $Q^{\alpha\mu\nu}=\int(p^\alpha-u^\alpha)(p^\mu-u^\mu)(p^\nu-u^\nu)fd\Omega$, where $u^\mu=J^\mu/h$ is the normalized hydrodynamic four momentum, $h=\int fd\Omega$ the invariant particle density, $J^\mu=\int p^\mu fd\Omega$ the fluid four current, and $d\Omega=d^3\mathbf{p}/p^0$ the Lorentz invariant momentum-space volume. Equation (1) implies the exact conservation laws

$$\partial_\mu(hu^\mu)=0, \tag{2}$$

$$hu^\mu\partial_\mu u^\nu+\partial_\mu\Theta^{\mu\nu}=(-e/mc^2)F^{\nu\alpha}hu_\alpha, \tag{3}$$

$$hu^\alpha\partial_\alpha(\Theta^{\mu\nu}/h)+\Theta^{\nu\alpha}\partial_\alpha u^\mu+\Theta^{\mu\alpha}\partial_\alpha u^\nu+\partial_\alpha Q^{\alpha\mu\nu}$$
$$=(-e/mc^2)(F^{\nu\alpha}\Theta^\mu_\alpha+F^{\mu\alpha}\Theta^\nu_\alpha), \tag{4}$$

which correspond to the continuity equation, energy-momentum conservation, and energy-momentum flux conservation, respectively. The inhomogeneous Maxwell equations are expressed as $\partial_\mu F^{\mu\nu}=4\pi\Sigma_s q_s J_s^\nu$, where the sum is over species with q the charge.

We will assume a "warm" plasma such that the distribution f has a small momentum spread about its mean [17–20]. We make no additional assumptions concerning the specific form of f. This warm assumption will allow the hierarchy of moment equations to be truncated. We define the invariant measure of thermal spread $\epsilon^2=-\Theta^\mu_\mu/h=u^\mu u_\mu-1$, where $|\epsilon|\ll 1$ is assumed, such that $\beta_{\rm th}^2=\epsilon^2(1+\epsilon^2)^{-1}\simeq\epsilon^2$ is the normalized thermal velocity spread (temperature). We will assume that in the local plasma rest frame $\Theta^{\mu\nu}/h=\mathcal{O}(\epsilon^2)$ and $Q^{\alpha\mu\nu}/h=\mathcal{O}(\epsilon^3)$. Truncation of the moment hierarchy to order $\mathcal{O}(\epsilon^2)$ is achieved by neglecting the third-order centered moment $Q^{\alpha\mu\nu}$ in the fluid equations. Note that ϵ is a Lorentz invariant and $\epsilon^2\ll 1$ is satisfied if the local rest frame temperature of the plasma is nonrelativistic. We consider the ratio $\lambda=n_p/h$ [17,19], where $n_p=(J^\mu J_\mu)^{1/2}$ is the proper density, and introduce $(\lambda\Gamma,\lambda\Gamma\mathbf{w})=u^\mu=J^\mu/h$, where $u^\mu u_\mu=\lambda^2$ and $\Gamma^{-2}=(1-\mathbf{w}\cdot\mathbf{w})$. Using the contraction of the energy-momentum stress tensor we find $\lambda^2=1-\Theta^\mu_\mu/h=1+\epsilon^2$, such that $\lambda=(1-\beta_{\rm th}^2)^{-1/2}$ is identified as the Lorentz factor associated with the thermal fluctuations.

Consider a plasma wave driven by a laser pulse propagating in the z direction with transverse normalized vector potential $\mathbf{a}_\perp=e\mathbf{A}_\perp/mc^2$ (Coulomb gauge). We consider one-dimensional motion such that $f=g(z,p_z,t)\delta^2(\mathbf{p}_\perp-\mathbf{a}_\perp)$ and the transverse component of Eq. (3) reduces to $u^\mu\partial_\mu(\lambda\Gamma\mathbf{w}_\perp-\mathbf{a}_\perp)=0$. For an initially quiescent ($\mathbf{w}=0$) plasma, $\lambda\Gamma\mathbf{w}_\perp=\mathbf{a}_\perp$, i.e., $\mathbf{w}_\perp=\mathbf{a}_\perp(1-w_z^2)^{1/2}(\gamma_\perp^2+\epsilon^2)^{-1/2}$, with $\gamma_\perp=(1+a_\perp^2)^{1/2}$. This is the generalization of canonical transverse fluid momentum conservation including thermal effects.

The contraction $g_{\mu\nu}Q^{\alpha\mu\nu}=0$ [to order $\mathcal{O}(\epsilon^2)$] implies $\Theta^{\mu0}=w_z\Theta^{\mu1}$, and $\lambda^2=1+(1-w_z^2)\Theta^{11}/h$. Equations (2)–(4) can be combined to yield

$$u^\mu\partial_\mu(h^{-3}\Gamma^{-2}\Theta^{11})=0. \tag{5}$$

For an initially quiescent plasma of density n_0, $\Theta^{11}/n_0=\Gamma^2(h/n_0)^3\theta$, where θ is the initial temperature normalized to $mc^2/k_{\rm B}$. Equation (5) is equivalent to a statement of entropy conservation.

Next, we assume the quasistatic approximation, such that the plasma wave driver (e.g., laser field or particle beam) and fluid quantities are functions only of $\xi=z-\beta_\varphi ct$. The continuity equation Eq. (2) becomes

$$\partial_\xi[h\lambda\Gamma(\beta_\varphi-w_z)]=0, \tag{6}$$

or, for an initially quiescent plasma of density n_0, $h=n_0[\lambda\Gamma(1-\beta_\varphi^{-1}w_z)]^{-1}$. The components of Eq. (3) can be combined to yield [using $\Theta^{\mu0}=w_z\Theta^{\mu1}$ and Eq. (6)]

$$\partial_\xi[(h\lambda^2\Gamma^2+\Theta^{11})(1-\beta_\varphi w_z)(1-\beta_\varphi^{-1}w_z)]=n_0\partial_\xi\phi, \tag{7}$$

where $\phi=e\Phi/mc^2$ is the normalized space-charge potential of the plasma wave and $(\lambda\Gamma)^2=(\gamma_\perp^2+\epsilon^2)/(1-w_z^2)$. Using Eqs. (5) and (6), Eq. (7) can be written as the following longitudinal constant of motion (conservation of energy in the wave frame):

$$\partial_\xi\left[\frac{\gamma_\perp(1-\beta_\varphi w_z)}{(1-w_z^2)^{1/2}}-\phi+\frac{3}{2}\theta\frac{(1-\beta_\varphi w_z)(1-w_z^2)^{1/2}}{\gamma_\perp(1-\beta_\varphi^{-1}w_z)^2}\right]=0. \tag{8}$$

The third term on the right-hand side of Eq. (8) is due to the energy in the thermal fluctuations (pressure).

The plasma wave potential is determined by the Poisson equation $c^2\partial_\xi^2\phi=\omega_p^2[J^0/n_0-1+n_b/n_0]$, where n_b/n_0 is the normalized density of a beam driver, $J^0/n_0=\lambda\Gamma h/n_0=\beta_\varphi/(\beta_\varphi-w_z)$, and the ions are assumed stationary. The Poisson equation can be combined with Eq. (8) to yield the evolution equation for the axial plasma fluid velocity w_z.

We consider the cases of plasma wave excitation behind a beam driver where $n_b(\xi)=0$, behind a short laser driver (e.g., the standard laser wake-field regime) where $\gamma_\perp=1$, and excitation under a long laser pulse (e.g., the self-modulated laser wakefield regime) where $\gamma_\perp^{-1}|(c/\omega_p)\partial_\xi\gamma_\perp|\ll 1$ and $\gamma_\perp\simeq$constant. Using Eq. (8), the first integral of the Poisson equation is (assuming $n_b=0$ and $\gamma_\perp=$constant) $\hat{E}^2=\gamma_\perp(\chi_0-\chi+\chi_0^{-1}-\chi^{-1})+[F(\chi_0)-F(\chi)]\theta/\gamma_\perp$, where $\chi^2=(1-w_z)/(1+w_z)$, $\hat{E}=E/E_0=-(c/\omega_p)\partial_\xi\phi(w)$,

$$F(\chi)=\frac{6\beta_\varphi^2\chi[(1-\chi^4)-\beta_\varphi(\chi^4-2\chi^2/3+1)]}{[(1-\beta_\varphi)-(1+\beta_\varphi)\chi^2]^3}, \tag{9}$$

and χ_0 corresponds to the momentum that produces the extremum of ϕ [i.e., $\hat{E}(\chi_0)=0$].

Solving $\partial_\xi\phi=0$ (i.e., a quartic equation for χ_0^2), yields the momentum that produces the extremum of ϕ,

$$\begin{aligned}\chi_0^2=&\ \gamma_\varphi^2(1-\beta_\varphi)^2+\frac{1}{2}\gamma_\perp^{-2}(1+\beta_\varphi)^{-2}\{3\beta_\varphi^2\theta\\&+\beta_\varphi(48\theta\gamma_\perp^2/\gamma_\varphi^2+9\beta_\varphi^2\theta^2)^{1/2}+[6\theta\beta_\varphi^2(10\gamma_\perp^2/\gamma_\varphi^2+3\beta_\varphi^2\theta)\\&+2\beta_\varphi(2\gamma_\perp^2/\gamma_\varphi^2+3\beta_\varphi^2\theta)(48\theta\gamma_\perp^2/\gamma_\varphi^2+9\beta_\varphi^2\theta^2)^{1/2}]^{1/2}\}.\end{aligned} \tag{10}$$

Equation (10) determines the fluid momentum at the maximum compression of the plasma. In the cold limit ($\theta=0$), $\chi_0^2=\gamma_\varphi^2(1-\beta_\varphi)^2$, and the extremum of the potential occurs when the axial fluid velocity equals the phase velocity of the wave, i.e., $w_z=\beta_\varphi$. In the ultrahigh phase velocity limit ($\beta_\varphi=1$), $\chi_0^2=3\gamma_\perp^{-2}\theta/2$.

Using the Poisson equation, the phase where $\hat{E}$ is maximum ($\partial_\xi\hat{E}=0$) occurs at the momentum $\chi=1$ (i.e., $w_z=0$). Evaluating $\hat{E}^2$ at $\chi=1$ yields

$$\hat{E}_{\rm max}^2=\gamma_\perp(\chi_0+\chi_0^{-1}-2)+[F(\chi_0)-1]\theta/\gamma_\perp, \tag{11}$$

where $F(\chi_0)$ is given by Eqs. (9) and (10). Equation (11) is the main result of this paper, and determines the maximum field amplitude E_{max} of a nonlinear plasma wave with phase velocity β_φ excited in a plasma with initial temperature θ. The maximum density perturbation is given by $(J^0/n_0)_{max} = [1-\beta_\varphi^{-1}(1-\chi_0^2)/(1+\chi_0^2)]^{-1}$. Note that the maximum plasma density perturbation in a warm plasma does not become singular, as in the cold fluid theory [5,6].

In the cold plasma limit ($\theta=0$), Eq. (11) reduces to $\hat{E}^2_{max}(\theta=0) = 2\gamma_\perp(\gamma_\varphi-1)$. This is a generalization of the cold relativistic wave-breaking field [5] to include a laser field. Note that $\hat{E}^2_{max}(\theta=0)$ is the same as the threshold field for trapping background plasma electrons in a cold plasma wave [21] (since the cold fluid element orbits are identical to the particle orbits).

For $\beta_\varphi \ll 1$, Eq. (11) reduces to

$$\frac{\hat{E}^2_{max}}{\gamma_\perp\beta_\varphi^2} \simeq 1 - \frac{8}{3}\left(\frac{3\theta}{\gamma_\perp^2\beta_\varphi^2}\right)^{1/4} + 2\left(\frac{3\theta}{\gamma_\perp^2\beta_\varphi^2}\right)^{1/2} - \frac{1}{3}\left(\frac{3\theta}{\gamma_\perp^2\beta_\varphi^2}\right), \tag{12}$$

where terms of order $\mathcal{O}(\theta\beta_\varphi^2)$ have been neglected. For $\gamma_\perp=1$, Eq. (12) is identical to the result of Ref. [7].

For $\beta_\varphi=1$ (e.g., an ultrarelativistic electron beam driver satisfying $\gamma_\varphi^{-2} \ll \theta \ll 1$), Eq. (11) reduces to

$$\hat{E}^2_{max} = \gamma_\perp^2 (2/3)^{3/2}\theta^{-1/2}[1-\gamma_\perp^{-1}(3\theta/2)^{1/2}]^3. \tag{13}$$

For the case $\gamma_\perp=1$, Eq. (13) scales to leading order as $E_{max} = \theta^{-1/4}\rho_{th}E_0$. Except for the factor $\rho_{th}\sim 1$, this scaling is the same as that obtained in Refs. [8,9].

In the limit, $\theta \ll \gamma_\perp^2/\gamma_\varphi^2 \ll 1$, Eq. (11) reduces to

$$\hat{E}^2_{max} \simeq 2\gamma_\perp(\gamma_\varphi-1) - 2\gamma_\varphi\left[\frac{4}{3}(3\gamma_\varphi^2\gamma_\perp^2\theta)^{1/4} - (3\gamma_\varphi^2\theta)^{1/2}\right]. \tag{14}$$

Equation (14) is the cold relativistic wave-breaking field (generalized to include a laser field) with the lowest order reduction due to the plasma temperature. For high-intensity lasers ($a_\perp \gtrsim 1$), Eq. (14) indicates that E_{max} inside a laser pulse is significantly larger compared to behind the pulse (where $a_\perp=0$). For a laser driver, the phase velocity of the plasma wave is approximately the nonlinear group velocity of the laser pulse, i.e., $\gamma_\varphi \simeq [\gamma_\perp(1+\gamma_\perp)/2]^{1/2}(\omega_0/\omega_p)$. Therefore, for ultrahigh intensities ($a_\perp \gg 1$), $\hat{E}_{max} \simeq (2\gamma_\perp\gamma_\varphi)^{1/2} \sim a_\perp(\omega_0/\omega_p)^{1/2}$ in the limit $\theta \ll \gamma_\perp^2/\gamma_\varphi^2 \ll 1$.

The transition from the laser-driven regime ($\gamma_\varphi^2\theta < 1$) to the ultrarelativistic beam-driven regime ($\gamma_\varphi^2\theta \gg 1$) is shown in Fig. 1, which plots $\hat{E}_{max}$ [Eq. (11)] vs $\gamma_\varphi^2\theta$ for $\theta=10^{-3}$, $\theta=10^{-4}$, and $\theta=10^{-5}$ with $\gamma_\perp=1$. The dashed lines in Fig. 1 are the $\beta_\varphi=1$ limit [Eq. (13)]. Note that for typical short-pulse laser-plasma-interactions, $\theta mc^2 \sim 10$ eV [13,14], or $\theta \sim 10^{-5}-10^{-4}$. Figure 1 shows the inaccuracy of using the ultrahigh phase velocity approximation ($\beta_\varphi=1$) in the laser-plasma accelerator parameter regime ($\theta\gamma_\varphi^2 < 1$).

The wavelength λ_{osc} of the nonlinear plasma oscillation at the maximum amplitude is computed from $\hat{E}$ by $\lambda_{osc} = \int d\xi = -2c\omega_p^{-1}\int(d\phi/d\chi)\hat{E}^{-1}d\chi$ between the extrema of χ. Figure 2 shows the wavelength of the plasma oscillation λ_{osc} at the maximum amplitude normalized to $\lambda_p = 2\pi c/\omega_p$ vs γ_φ for initial temperatures $\theta=10^{-3}$, $\theta=10^{-4}$, and $\theta=10^{-5}$, with $\gamma_\perp=1$. The dashed line in Fig. 2 shows λ_{osc}/λ_p for an initially cold plasma $\theta=0$.

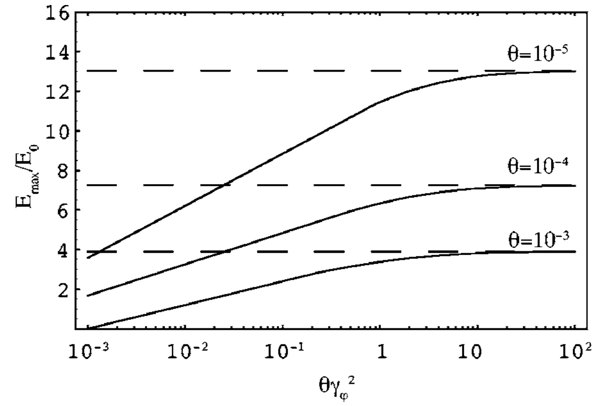

FIG. 1. Maximum plasma wave electric field $\hat{E}_{max} = E_{max}/E_0$ [Eq. (11)] vs $\theta\gamma_\varphi^2$ for initial temperatures $\theta=10^{-3}$, $\theta=10^{-4}$, and $\theta=10^{-5}$, with $\gamma_\perp=1$. Dashed lines are the ultrahigh phase velocity result $\beta_\varphi=1$ [Eq. (13)].

The temperature (thermal velocity spread) evolution is given by $\beta_{th}^2 = \epsilon^2 = \theta(1-w_z^2)\Gamma^2(h/n_0)^2$, which is maximum at the maximum compression of the plasma ($\chi=\chi_0$), i.e., $\epsilon^2_{max} = 4\chi_0^2[(1+\chi_0^2)-\beta_\varphi^{-1}(1-\chi_0^2)]^{-2}\theta$. For an ultrarelativistic beam driver ($\beta_\varphi=1$ and $\gamma_\perp=1$), $\epsilon^2_{max}=2/3$ [the upper bound of $\epsilon^2_{max}(\beta_\varphi)$]. In the limit $\theta \ll \gamma_\perp^2/\gamma_\varphi^2 \ll 1$ (e.g., laser driver), the maximum temperature is, to leading order, $\epsilon^2_{max} \simeq \gamma_\perp(\gamma_\varphi^2\theta/3)^{1/2}[1-(3\gamma_\varphi^2\theta)^{1/2}/(4\gamma_\perp)] \ll 1$, which confirms the validity of the warm plasma approximation $\epsilon^2 \ll 1$.

The above results for $\hat{E}_{max}$ are independent of the driver. Consider excitation by a laser pulse with length optimized to maximize the wave amplitude. As the laser intensity increases, the wave amplitude increases, up to the amplitude at which $|\hat{E}| = \hat{E}_{max}$, which is first reached behind the laser pulse (where $\gamma_\perp=1$). Note that the maximum density compression occurs at the phase where $\hat{E}=0$, which is at a phase behind that where $|\hat{E}| = \hat{E}_{max}$ in a warm plasma. Physically, the limit

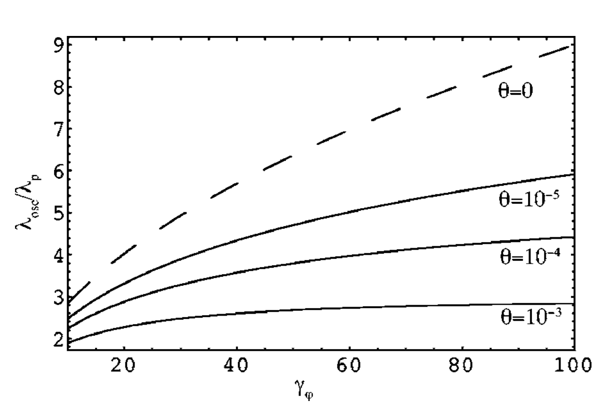

FIG. 2. Normalized nonlinear plasma wavelength λ_{osc}/λ_p at the maximum plasma wave amplitude vs γ_φ for initial temperatures $\theta=10^{-3}$, $\theta=10^{-4}$, and $\theta=10^{-5}$, with $\gamma_\perp=1$. The dashed line is the cold limit $\theta=0$.

on the wave amplitude is due to the pressure force. As the plasma becomes highly compressed, the pressure force grows, ultimately limiting the density compression and therefore the wave amplitude. This is in contrast to cold fluid theories where the maximum field is reached when the density becomes singular (and shock formation occurs). For larger drive intensities, no force balance is possible, and no traveling wave solutions exist.

For a laser with a square pulse profile, the maximum amplitude is obtained when the laser pulse length is of an optimal value such that $\partial_\xi \phi = 0$ at the end of the laser pulse. Note that, for an optimal length driver, the laser initially reduces the plasma density and the pressure force will remain small during the excitation of the plasma wave by the laser pulse. For relativistic plasma waves ($\gamma_\varphi^2 \gg 1$), the laser intensity required to excite the maximum field Eq. (11) is $\gamma_\perp \simeq \hat{E}_{\max}/2 + [(\hat{E}_{\max}/2)^2 + 1]^{1/2}$. The limits $\gamma_\varphi^2 \gg 1$ and $\gamma_\varphi^2 \theta \ll 1$ imply $\gamma_\perp \simeq \sqrt{2\gamma_\varphi}[1 - (2^{3/2}/3)(\gamma_\varphi^2\theta/2)^{1/4} + (5/9)(\gamma_\varphi^2\theta/2)^{1/2}]$.

In this paper a comprehensive theory has been presented that describes the properties of nonlinear electron plasma waves with arbitrary phase velocity in a warm plasma, including the presence of an intense laser field. An analytical result for the maximum field amplitude is derived, Eq. (11). Equation (11) is capable of describing the regime of current ultraintense short-pulse laser interactions with underdense plasma, in contrast to previous results that are limited to ultrarelativistic particle drive beams. The maximum field is larger in the presence of an intense laser field. These results place a fundamental limit on the accelerating gradient in plasma-based accelerators.

The authors gratefully acknowledge discussions with A. Charman. This work was supported by the Director, Office of Science, Office of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231.

[1] E. Esarey *et al.*, IEEE Trans. Plasma Sci. **24**, 252 (1996).
[2] S. P. D. Mangles *et al.*, Nature (London) **431**, 535 (2004).
[3] C. G. R. Geddes *et al.*, Nature (London) **431**, 538 (2004).
[4] J. Faure *et al.*, Nature (London) **431**, 541 (2004).
[5] A. I. Akhiezer and R. V. Polovin, Zh. Eksp. Teor. Fiz. **30**, 915 (1956) [Sov. Phys. JETP **3**, 696 (1956)].
[6] J. M. Dawson, Phys. Rev. **113**, 383 (1959).
[7] T. P. Coffey, Phys. Fluids **14**, 1402 (1971).
[8] T. Katsouleas and W. B. Mori, Phys. Rev. Lett. **61**, 90 (1988).
[9] J. B. Rosenzweig, Phys. Rev. A **38**, 3634 (1988).
[10] A. Modena *et al.*, Nature (London) **377**, 606 (1995).
[11] D. Umstadter *et al.*, Science **273**, 472 (1996).
[12] W. P. Leemans *et al.*, Phys. Rev. Lett. **89**, 174802 (2002).
[13] C. G. Durfee, III, J. Lynch, and H. M. Milchberg, Phys. Rev. E **51**, 2368 (1995).
[14] P. Volfbeyn, E. Esarey, and W. Leemans, Phys. Plasmas **6**, 2269 (1999).
[15] J. B. Rosenzweig *et al.*, Phys. Rev. Lett. **61**, 98 (1988).
[16] S. R. de Groot, W. A. van Leeuwen, and C. G. van Weert, *Relativistic Kinetic Theory* (North-Holland, Amsterdam, 1980).
[17] W. A. Newcomb, Phys. Fluids **25**, 846 (1982).
[18] P. Amendt, Phys. Fluids **29**, 1458 (1986).
[19] J. G. Siambis, Phys. Fluids **30**, 896 (1986).
[20] B. A. Shadwick, G. M. Tarkenton, and E. H. Esarey, Phys. Rev. Lett. **93**, 175002 (2004).
[21] E. Esarey *et al.*, Phys. Plasmas **6**, 2262 (1999).